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COMMENT

# Comment on 'Scaling in reaction–diffusion systems'

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**Abstract.** It is shown that the claims and results of Elderfield and Wilby on critical behaviour in reaction-diffusion models of the form $pX \leftarrow mX \rightarrow qX$, $nX \rightarrow sX$ are incorrect on a number of points.

In the letter of Elderfield and Wilby (1987) the critical behaviour in reaction-diffusion models of the form $pX \leftarrow mX \rightarrow qX$, $nX \rightarrow sX$ is studied. These reactions are characterised by positive integers with $m < n$, $p < m < q$, $s < n$. Using field-theoretic renormalisation group methods the authors claim that each different pair $(m, n)$ constitutes a new universality class in the spirit of critical phenomena. For the specific values $m = 1$, $n \geqslant 3$ and $m = 2$ they calculate critical exponents and vertex functions respectively. However, the claims and results of Elderfield and Wilby (1987) are incorrect on a number of points.

(i) They derive a reduced Lagrangian

$$L_{\pm} = \int \mathrm{d}^d x \{ \mathrm{i}\hat{\alpha}[(\partial/\partial t) - D_0 \nabla^2 \alpha - r_0 \alpha^m] + g_0(\mathrm{i}\hat{\alpha}\alpha^n \pm \hat{\alpha}^2 \alpha^m) \} \tag{1}$$

and try to renormalise it by introducing several $Z$-factors which are determined by minimal subtraction of the dimensionally regularised diagrams. However, the Lagrangian is non-renormalisable for $n \geqslant 3$. This is easily seen by power counting (e.g. vertex functions $\Gamma_{1,k}$ with one $\hat{\alpha}$-leg and $k < n$ $\alpha$-legs are primitively divergent). These divergences cannot be detected if one uses the dimensionless regularisation method carelessly. This procedure only determines the logarithmic divergences and neglects all additive renormalisations needed for this Lagrangian.

(ii) The correct treatment of the Lagrangian (1) starts with a finite cutoff (naturally given by atomic units) and uses a Wilson-like renormalisation procedure. A single coarse graining step and the following rescaling suffice to generate a set of new interactions, especially of the type $\mathrm{i}\hat{\alpha}\alpha^k$ with $k = 1, \ldots, n-1$ and $\hat{\alpha}^2\alpha^l$ with $l = 1, \ldots, m-1$. The most relevant of these are $\mathrm{i}\hat{\alpha}\alpha$, $\mathrm{i}\hat{\alpha}\alpha^2$, $\hat{\alpha}^2\alpha$ which control the fixed-point behaviour. Thus all the seemingly different models belong to the universality class of the Schlögl model (Schlögl (1972), see, e.g., Grassberger and de la Torre (1979), Janssen (1981) and literature cited therein) with $m = 1$, $n = 2$.

(iii) The physical origin of the fact that all one-species reaction-diffusion models with an absorbing vacuum state belong to the Schlögl universality class can be traced back to the strong cluster properties of the reacting and diffusing particles seen, e.g., in the computer simulations (Grassberger and de la Torre (1979), Grassberger (1982) and private communications). Clusters act as big particles if viewed on a coarser scale.

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They dissociate and annihilate as in the Schlögl reaction scheme irrespective of the specific 'microscopic' reaction.

(iv) In more complex one-species reaction schemes with an absorbing vacuum state a discontinuous transition can arise in addition to a continuous transition, depending on the domain of parameter values. These two types of transition are separated by a tricritical point. The universal behaviour near that point can be described by a careful analysis of the Lagrangian with $m = 1$; $n = 3$ supplemented by appropriate counterterms (such a model is similar to the $\Phi^6$-model for thermal tricritical points). In this respect the calculation of the critical exponents made by Elderfield and Wilby (1987) could be of some use. However, their numerical calculations are also in error. The correct results are found in the work of Ohtsuki and Keyes (1987) which unfortunately also suffers from non-renormalisabilities and incorrect treatment of the Lagrangian.

(v) In the case $m = 2$ Elderfield and Wilby (1987) claim to have calculated vertex functions 'exactly'. However, they neglect many diagrams; e.g., for $k = 3$ the simplest one is of the structure shown in figure 1. Thus here the calculation is in error as well. The influence of the neglected interactions in the unstable Lagrangian $L_-(1)$ is uncontrollable and the calculation leads to dubious results.

Elsewhere I shall give a correct treatment of tricritical behaviour in one-component reaction-diffusion systems with an absorbing vacuum state.

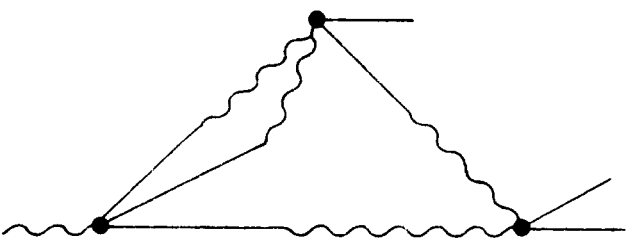

**Figure 1.**

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